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Radiation Pattern for a Multiple-Element HFGW Generator

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Abstract. We calculate the values for the High-Frequency Gravitational Wave (HFGW) radiation pattern for a multiple-element HFGW generator in the "far field," that is the field many wavelengths away from the generator. We extend Eqs. (8) and (9) from Baker, Davis and Woods (2005) for a single GW-emission pair to include an in-phase, linear array of N such pairs as discussed in Baker, Stephenson and Li (2008). We calculate new values for variable K from Eq. (8) of Baker, Davis and Woods (2005) by decreasing the integration interval of θ from 10° to 1°. This provides us with a K value of increased accuracy. The improved K has a value of $7.6\text{x}10^{-7}$ deg^{-2} and is then used in Eq (9) of Baker, Davis and Woods (2005) to find the power intensity, I(θ), of a single GW source in terms of watts per square degree over the radiation-pattern cap The θ half-power-point angle for a single GW-emission pair at their mid-way-point focus is also recalculated and found to be 47.5°. We utilize the result of Romero and Dehnen (1981) and Dehnen and Romero-Borja (2003) for an increase in HFGW flux (in a linear array of N in-phase radiation elements) proportional to N^2. This result is employed to compute the half-power-point angle, idealized radiation cap area and the HFGW flux/power-of-a-single-radiation-element at a distance of several wavelengths away, for example one meter from the end of a linear and a double-helical array in Wm^{-2} as a function of N. The notional picture shown of an idealized needle-like radiation beam is in the far field. It is described at a distance far enough from the generator that it is beyond the conventional diffraction limit of a beam's radiation-pattern cap area. It is found that the HFGW flux calculated is small, but that the Li-Baker detector may be capable of sensing the HFGWs generated in a laboratory setting.

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INTRODUCTION

Papers by Romero and Dehnen (1981), Dehnen and Romero (2003), Woods and Baker (2005), Baker, Woods and Li (2006) and by Baker, Stephenson and Li (2007) have analyzed the generation of high-frequency gravitational waves (HFGWs) by means of a linear array of piezoelectric crystals energized in step with the passage of the HFGW. The Dehnen and Romero papers (which utilized classical General Relativity techniques) provided results that were in agreement with the other references and showed that if there were N such piezoelectric elements in a linear array, then the HFGW flux is proportional to N squared. Baker, Davis and Woods (2005) analyzed the radiation pattern for a single pair of piezoelectric crystals (an array radiation element) or two clusters of such crystals based upon the classical analyses of Landau and Lifshitz (1975). The concept of the GW generators can be visualized by utilizing the orbital model of Landau and Lifshitz (1975) and the orbital spiral shown in Fig. 2-23, p. 214 of Baker (1967). Consider a stack of orbiting masses. That is, a stack of orbital planes one on top of the other. Each produces GWs centered about the masses' common center. As the GWs move up these centers a GW flux grows or accumulates in direct proportion to the square of the number of orbital planes. We accomplish an analytical determination of an idealized radiation pattern of a linear array of N such radiation elements, each having the generated GW power in watts of P_i. The notional picture shown in Fig. 1 of an idealized needle-like radiation beam is in the far field. It is described at a distance of many wavelengths, far enough from the generator that it is beyond the conventional diffraction limit of a beam's radiation-pattern cap area. A double-helix array of such radiation elements is examined that provides a convenient means to generate HFGWs.

PARAMETER IMPROVEMENT

We calculate new values for variable K from Eq. (8) of Baker, Davis and Woods (2005) by decreasing the integration interval of θ from 10° to 1°. This provides us with a K value of increased accuracy. The K has a value of $7.6x10^{-7}$ deg^{-2} and is then used in Eq (9) of Baker, Davis and Woods (2005) to find the power intensity, I(θ), of a single GW source in terms of watts per square degree over the radiation-pattern cap The θ half-power-point angle for a single GW-emission pair at their mid-way-point focus is also recalculated and found to be 47.5°. We utilize the result of Romero and Dehnen (1981) and Dehnen and Romero-Borja (2003) for an increase in HFGW flux (in a linear array of *N* in-phase radiation elements) proportional to N^2. (Essentially, one takes the sum of the ever growing HFGW flux (Wm^{-2}) in the beam as $\Sigma N = N(N+1)/2 \sim N^2$.) This result is employed to compute the half-power-point angle, idealized radiation cap area and the HFGW flux at a at a distance of several wavelengths away, for example one meter from the end of an array in Wm^{-2} as a function of *N*. The following Fig. 1 exhibits notional drawings of the three-dimensional radiation pattern of a single (*N*=1) GW radiation element (focused midway between two jerked masses) Fig. 1 (a), of the cross-section of the radiation pattern and the half-power-point angle $\theta_{1/2(N=1)}$ Fig. 1(b) and the single-point (*N*=1) radiation pattern superimposed on the idealized needle-like radiation pattern of a linear array of twenty such elements (*N*=20) excited in step with the advancing and growing HFGW front, Fig. 1 (c).

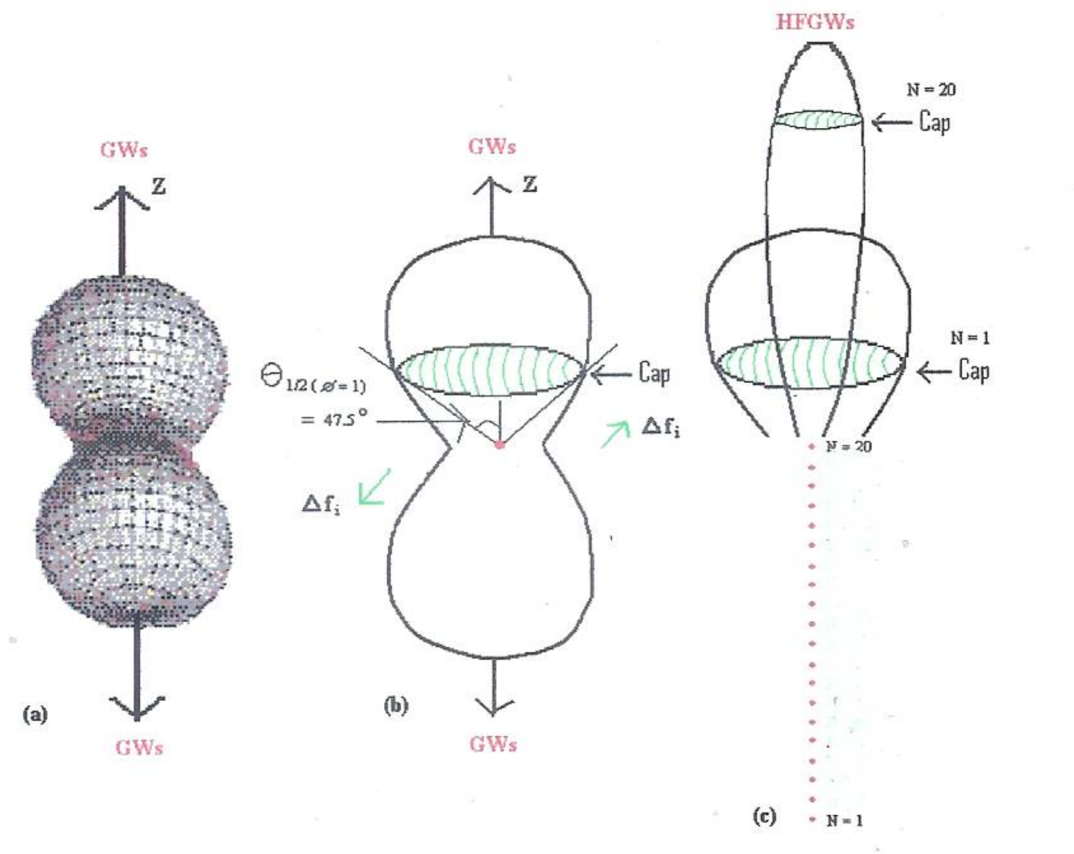


FIGURE 1. (a) Radiation Pattern of Two Jerking Masses on Orbit (Landau and Lifshitz (1975). (b) The Radiation Pattern When $N = 1$. (c) A Notional Drawing of the Idealized Radiation Pattern with $N = 1$ and $N = 20$.

DEVELOPMENT OF THE FAR-FIELD RADIATION PATTERN WITH NUMBER OF RADIATING ELEMENTS *N*

We visualize the radiation pattern of any single gravitational wave or GW radiation element pair ($N = 1$) as having a peanut shape (it is generated at the midpoint between two jerked masses – that is, one radiation element) as defined by Landau and Lifshitz (1975).. In a linear array of *N* such elements all the axes of peanuts are aligned end to end, but we look at a single element's ($N = 1$) radiation pattern first. The top "hemisphere" (or "hemispeanut") of the radiation pattern has half of the radiator element's power spread out over its surface. The other half radiates in the opposite direction and in the opposite hemisphere, but it is not in phase as the backward-moving GW moves out. Only the forward-moving GW accumulates as each element is excited in turn as the GW move along in step with the excited array elements. For an individual excited GW radiation element ($N = 1$) the "half-power point" angle is defined as the semi-vertex angle to the point on the radiation pattern where the power intensity within the cone defined by this angle is half (1/2) of the radiation element's power in the upper hemisphere where half the GW power is radiated. That is, the half-power-point angle, $\theta_{½(N=1)}$, is the angle to the point where the summation (or integration) of Eq. (8) of Baker, Davis and Woods is one-forth of a watt for a one-watt radiation element power. We solve for $\theta_{½(N=1)}$ by trial and error. The area of the half-power-point-angle cap for $N = 1$, on a *unit one square meter sphere* having radius 0.282 m, is

$$A_{½(N=1)}\ (0.282) = \pi[\sin(\theta_{½(N=1)})(0.282)]^2 = 0.1358\ \text{m}^2. \qquad (1a)$$

and at one meter it is

$$A_{½(N=1)}\ (1.0) = A_{½(N=1)}\ (0.282)/(0.282)^2 = 1.7608\ \text{m}^2\ . \qquad (1b)$$

Our hypothesis for a linear array of *N* GW radiators is based upon the analysis by Dehnen and Borja (1981, 2003) that shows that the GW flux (watts per square meter) of a linear array of GW emitters (that are in phase with the generated, moving gravitational wave front) increases as the square of the number of elements in the array, *N*. One N multiplier can be thought of as coming from the fact that *N* radiators or individual GW generators have *N* times the power of any given single radiator. The other *N* multiplier can be thought of as coming from the focusing effect of the linear array that produces a needle-like (not peanut-like) radiation pattern whose radiation-pattern area reduces according to $1/N$ at a distance of several wavelengths from the generator or "far field." Thus the actual GW flux Wm^{-2} increases with N^2. For $N > 1$, the area of the idealized cap decreases and is $A_½/N$. In order to define half-power point angle, $\theta_½$, for $N>1$, we set the cap area as

$$A_{cap} = A_{½(N=1)}/N \qquad (2)$$

and for a one-meter-squared area sphere (radius = 0.282 m) for $N = 1$

$$A_{cap} = \pi[\sin\theta_½\,(0.282)]^2 = A_{½\ (N=1)}\ . \qquad (3)$$

For the computed value of $\theta_½ = 47.5^0$, $A_{½\ (N=1)} = 0.1358\ \text{m}^2$. Thus for $N > 1$ the half-power point angle, $\theta_½$, is given by

$$\theta_½ = \sin^{-1}(\,[\sqrt{A_{½(N=1)}/N\pi}]/0.282) \text{ or} \qquad (4a)$$

$$\theta_½ = \sin^{-1}(0.737/\sqrt{N}). \qquad (4b)$$

For $N = 20$, $\theta_½ = 9.489^0$ and the area of the half-power-point of the idealized needle-radiation pattern is, from Eq. (2) $= 0.1358/\sqrt{20} = 0.0304\ \text{m}^2$ on the unit-area (1 m^2) sphere. The flux (Wm^{-2}) at the surface of this unit-area sphere or a distance of 0.282 meters from the end of the HFGW-generator array, over the area intercepted by the half-power-point cone is

$$F\ (0.282) = P_i N(1/4)/(\pi\ (\sin^2(\theta_½)))\ \ \text{Wm}^{-2} \qquad (5a)$$

So that for $N = 1$ $F(0.282) = P_i\, 0.14639\ P_i\ \ \text{Wm}^{-2}$ and at a distance of, for example, one meter it is

$$F(1.0) = F\ (0.282)/(1/0.282)^2. \qquad (5b)$$

By combining Eqs. (5a) and 5(b) we have for the half-power point GW flux 1.0 meters from the end of the array in which all of the radiation elements are aligned "end to end" that is, the radiating pairs of jerking mass elements are opposite to each other across the z axis in a plane perpendicular to the z axis shown in Fig. 1 and jerk in opposite directions

$$F\ (1.0) = (6.23\text{x}10^{-3})P_i\, N/(\ (\sin^2 (\theta_{½}))\ \ \text{Wm}^{-2} \qquad (5c)$$

where P_i is the power of any i^{th} radiation element pair. For N=1 the flux F(1.0) = (0.01146) P_i Wm^{-2} and for N = 20 the flux $F(1.0) = (20)^2F(1.0)_{N=1}$ = 4.58 The half-power-point angle, idealized beam cross-section area and flux as a function of N is presented in Table 1. In general, in the far field (many wavelengths distant from the HFGW generator array):

$$F(1.0) = N^2F(1.0)_{N=1}\ .= N^2\ (0.01146)\ P_i \quad . \qquad (6)$$

As examples consider a 5 GHz HFGW having λ = 6 cm and N = 100. From Eq. (4b) $\theta_{½} = 3.9^0$ or 0.068 radians, A_{cap} for unit area sphere =$1.151\text{x}10^{-3}$ m^2 from Eq. (5c) and the flux one meter from this point = 114.6 per P_i. Wm^2. For $N = 1.6\text{x}10^8$, $\theta_{½}$ = $3.3\text{x}10^{-3}$ degrees or $5.8\text{x}10^{-5}$ radians and flux = $2.9\text{x}10^{14}$ per P_i . Next consider Infrared (IR) energizing ring HFGW generator suggested by Woods and Baker (2009), in which λ = 50,000 $A^0/2$ or $2.5\text{x}10^{-6}$ m and if $N = 1.70 \text{ x } 10^{12}$. $\theta_{½}$ = $2.48\text{x}10^{-11}$ degr. or $4.3\text{x}10^{-13}$ rad., A_{cap} for unit area sphere = $8\text{x}10^{-14}$ m^2 and the flux for an N^2 law 1 m from this point = $3.3\text{x}10^{22}$ per P_i Wm^{-2}. As will be seen, however, for the IR case the square law only relates to n "plates" of rings and = 10^6.

TABLE 1. Half-Power-Point Angle, Beam Cross-section Area and Flux as a Function of N.

N	$\theta_{1/2}$, Degrees	$\theta_{1/2}$, Radians	A_{cap} for unit area sphere, m	Flux/P_i @ 1m, Wm^{-2} (when x P_i)
1	47.5	0.829	0.1358	0.01146
20	9.489	0.1656	$6.79\text{x}10^{-3}$	4.58
100	4.228	0.0738	$1.358\text{ x}10^{-3}$	114.6
10^3	1.3359	0.02331	$1.358\text{ x}10^{-4}$	$1.146\text{x}10^4$
10^4	0.4224	$7.372271\text{ x}10^{-3}$	$1.358\text{ x}10^{-5}$	$1.146\text{ x}10^6$
10^5	0.1336	$2.33176\text{ x}10^{-3}$	$1.358\text{ x}10^{-6}$	$1.146\text{x}10^8$
10^6	0.04224	$7.37227\text{ x}10^{-4}$	$1.358\text{x}10^{-7}$	$1.146\text{x}10^{10}$
10^7	0.01336	$2.33176\text{ x}10^{-4}$	$1.358\text{x}10^{-8}$	$1.146\text{x}10^{12}$
$1.6\text{x}10^8$	0.00338	$5.8\text{x}10^{-5}$	$8.49\text{x}10^{-10}$	$2.9\text{x}10^{14}$
$1.7\text{x}10^{12}$	$2.48\text{x}10^{-11}$	$4.3\text{x}10^{-13}$	$8\text{x}10^{-14}$	$3.3\text{x}10^{22}$
Avogadro's 6.02×10^{23}				$4.6\text{x}10^{45}$

For a ten-degree cone intercepting the radiation pattern, at a location which is nearer the end of the beam, the flux is larger as exhibited in Table 2. The ten-degree cap was originally suggested in Baker, Davis and Woods (2005) to better portray the larger flux at the idealized needle-like beam's end. The area of the ten-degree cap for N = 1 is $A_{10^0\,cap} = \pi[\sin\ (10^0)\ (0.282)]^2$ = $7.53\text{x}10^{-3}$ m^2 and for N = 20 it is $7.53\text{x}10^{-3}/20 = 3.75\text{x}10^{-4}$ m^2. The semi-vertex angle to the notional ten-degree cap at the top of the needle radiation pattern is proportional to the radius of the cap there or inversely proportional to √N and is also shown in Table 1. The flux at the needle cap (Wm^{-2}) at a distance of 0.282 meters from the end of the HFGW-generator array is

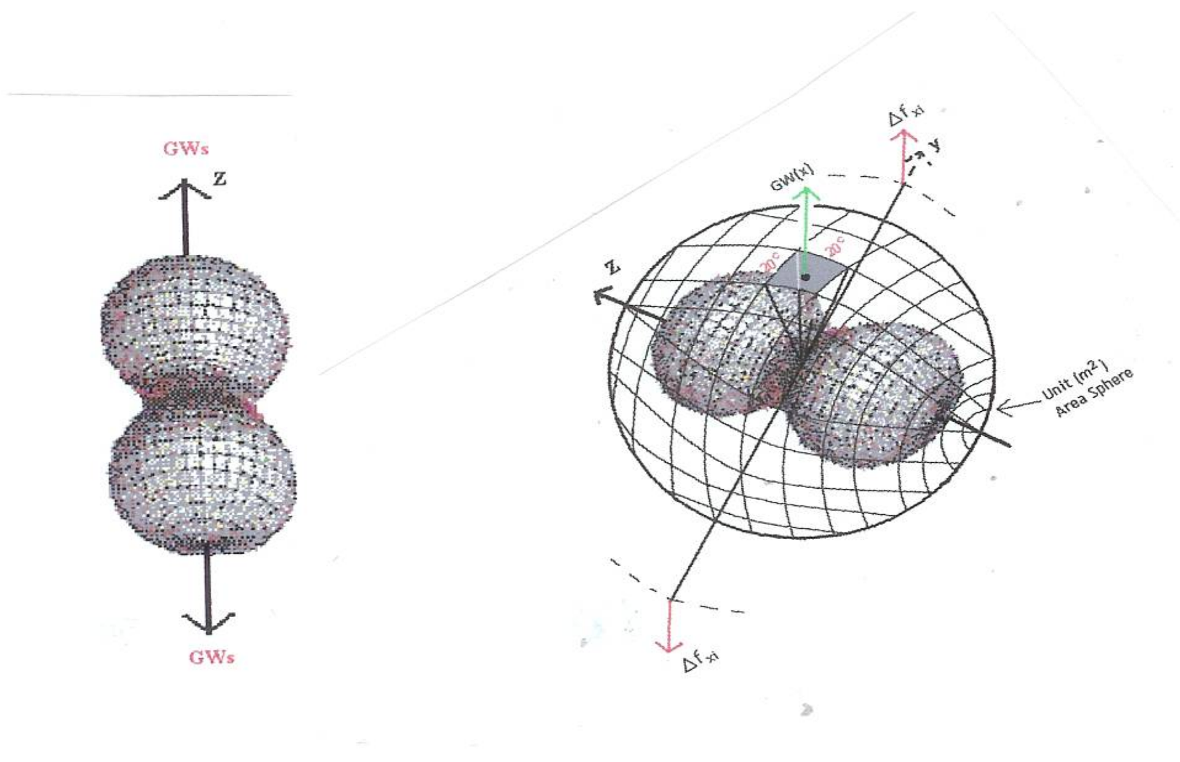
$$F\ (0.282) = \ P_i\,N\text{x}\ 3.183\text{x}10^{-2}/\ A_{needlecap} \quad \text{Wm}^{-2} \qquad (7)$$

where, by numerical integration using a one-degree interval, the power radiated by a one-watt source over a unit-area (1 m^2) sphere (radius of 0.282 m), in the ten-degree cone is $3.183\text{x}10^{-2}$ watts at a distance of 0.282 meters and the flux there is 4.227 Wm^{-2} So that at a distance of one meter it is $(4.227)(0,282)^2 = 0.336\ \text{Wm}^{-2}$.

TABLE 2. For a Ten-Degree Half Angle, Beam Cross-section Area and Flux as a function of N.

N	**$A_{needlecap}$ for unit area sphere, m^2**	**Flux/P_i @ 1 m, Wm^{-2} (when x P_i)**
1	$7.53x10^{-3}$	0.336
20	$3.75x10^{-4}$	13.44
100	$7.53x10^{-5}$	336
10^3	$7.53x10^{-6}$	$3.36x10^4$
10^4	$7.53x10^{-5}$	$3.36x10^6$
10^5	$7.53x10^{-6}$	$3.36x10^8$
10^6	$7.53x10^{-7}$	$3.36x10^{10}$
10^7	$7.53x10^{-8}$	$3.36x10^{12}$
10^8	$7.53x10^{-9}$	$3.36x10^{14}$
10^9	$7.53x10^{-10}$	$3.36x10^{16}$

With regard to the IR-generated HFGW, consider the radiation pattern of two masses (submicroscopic) on opposite sides of a ring. If the equal and opposite delta forces (or jerks) on the masses are in the plane of the ring and normal to a line between the two masses (as in the case of the centrifugal-force delta forces acting on orbiting masses considered by Landau and Lifshitz (1975)), then the radiation pattern is as shown in Fig. 2(a). If, on the other hand, the delta forces or jerks act perpendicular to the plane of the ring, as in the case of the standing-wave HFGW generator described by Woods and Baker (2009), then the peanut-shaped radiation pattern will have its major axis in the plane of the ring and perpendicular to the line connecting the two masses at its mid point as shown in Fig. 2(b). Consider a central band at the waist of the latter radiation-pattern peanut that is ± 10^0 from the midpoint or waist of the peanut. The power radiated from this band is one watt minus twice the power through each of two 80^0 cones in each hemisphere. By numerical integration (1^0 integration interval) the power radiated from this central band or belt is 1 – 2x(0.486) = 0.028 watts. If we consider a 20^0 section or wedge from this band, then the power passing through this 20^0 by 20^0 section of the radiation pattern is 0.028x(20/360) = 0.001555 watts. The area of this 20^0 by 20^0 section of the one-square-meter unit sphere is $(20x20)/4\pi(57.3)^2 = 0.0097$ m^2 . Thus the GW flux passing through this 20^0 by 20^0 section from a one-watt source at the surface of the unit-area (1 m^2) sphere is 0.001555/0.0097 = 0.16 Wm^{-2} and at a distance of one meter it is $(0.16)(0.282)^2 = 0.01272$ Wm^{-2}. Table 3 provides the relevant information.



(a) (b)

FIGURE 2. (a) The Equal and Opposite Delta Forces (or Jerks), Δf s, on the Masses are in the Plane of the Ring and Act Normal to a Line Between the Two Masses or Δf s Tangent to Helix Ribbons, $\pm \Delta f x_x$ (b) The Delta Forces or Jerks Act Perpendicular to the Plane of the Ring, as in the Case of the Standing-Wave IR HFGW Generator GW Radiation Build Up, $\pm \Delta f z_x$

For ***N*** jerkable masses, as in the case of an IR ring, Eq.(6) for the flux one-meter from the array end becomes

$$F(1.0) = N F(1.0)_{N=1} \;.= N\,(0.01146)\, P_i \quad . \tag{8}$$

TABLE 3. For a Twenty-Degree x Twenty-Degree Wedge and Transverse Δf s ,GW N^2 Flux as a function of *N*.

N	**Flux/P_i @ 1 m, Wm^{-2} (when x P_i)**
1	0.01272
20	5.09
100	127.2
10^3	1.27×10^4
10^4	1.27×10^6
10^5	1.27×10^8
10^6	1.27×10^{10}
10^7	1.27×10^{12}
2.1×10^{21}	1.27×10^{14}
Avogadro's Number 6.02×10^{23}	4.6×10^{45}

In order to accumulate and build up a GW wave front (flux) in proportion to N^2, another pair of jerking elements must be excited and create another GW source exactly when the GW wave front from the first source reaches it. If there are two parallel tracks of jerking sources (as in Baker, Stephenson and Li, 2007), then the energizing signal simply goes along each track in unison at the speed of light (speed of both EM and GW) and the GW sources build up the GW as they should. Now consider a double helix as in Fig. 3 (Patent Pending). Connect each element (jerking mass, e.g., Δf_x) of one of the helixes with the one immediately above it on the same helix. One can visualize this as a "cage" of vertical lines. Each vertical line is essentially a "track" and one track on one helix is parallel to and exactly opposite to a track on the other helix (2 r apart, where r is the helix radius). Just as in the case of the double-FBAR-track HFGW generator, it is only necessary to energize the elements up the tracks (or up the common z-axis of the helixes) by an energizing EM wave moving up the axis at the speed of light. In this case a powerful microwave beam can be projected up the axis of the helixes to excite or energize the jerking elements (e.g., FBARs of Baker, Woods and Li, 2006) in sequence at the speed of light. The energizing beam can be created by generators lined up on the axis of the helixes since the HFGW beam there will not be affected by their presence). Like the parallel-track FBAR configuration of Baker, Stephenson and Li (2008) there can be staggered rows of FBARs on the helix ribbons in order to greatly reduce their length and the helixes, size.

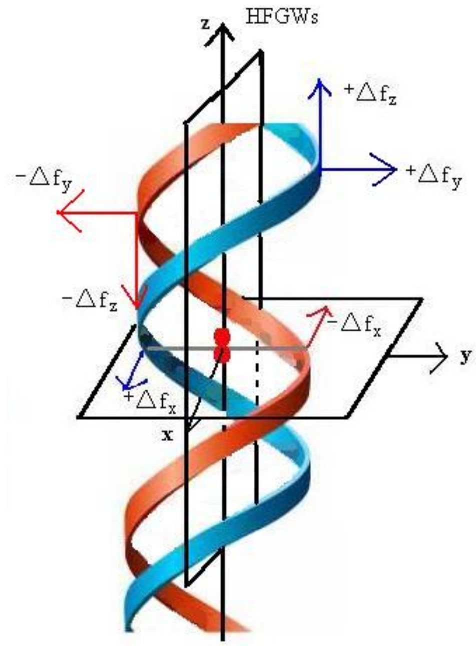


FIGURE 3. Double Helix Configuration (Patent Pending)

If the helix ribbons are tracks of FBARs the result is as follows: From Eq. (2) of Baker, Woods and Li (2006)

$$P(r, \Delta f, v) = 1.76\times10^{-52}(2r\, v_{GW}\, \Delta f)^2 \text{ W}. \quad (9)$$

So that for r=100 m, $v_{GW} = 4.9\text{x}10^9$ s^{-1} and $\Delta f = 2$ N (for each individual FBAR), $P_i = 6.76\text{x}10^{-28}$ W. We find the HFGW flux for $1.6\text{x}10^8$ FBARs is from Table 1 = $(2.9\text{x}10^{14})(6.76\text{x}10^{-28}) = 1.96\text{x}10^{-13}$ Wm^{-2}. If the radius of the helixes was dropped to a wavelength (6 cm), then the P_i would be reduced to $(0.06/100)^2(1.96\text{x}10^{-13}) = 1.18\text{x}10^{-16}$ Wm^{-2}.

There is significant promise for the IR-generated HFGWs suggested by Woods and Baker (2009). If you have a standing wave in a waveguide ring and excite it properly, then you have a GW source at its center (Patent Pending). The GW flux produced at its center is proportional to the *N* submicroscopic particle pairs in the ring. There is no N^2 build up but there is a *N* build up. If you have a stack of *n* rings, which are excited in sequence at light speed as a generated, growing GW passes by, then you have a n^2 build up in GW flux. But what we would like is a N^2 build up in order to take advantage of the enormous number of submicroscopic massive particles. We conjecture that one might be able to take advantage of the fact that it is only necessary to "light up" or create a GW source as the GW passes. For example, if at one location a GW source is created every $(1/3)\text{x}10^{-12}$ seconds and it is $[(1/3)\text{x}10^{-12}][3\text{x}10^{8}\ \text{ms}^{-1}) = 10^{-4}$meters away from an adjacent source location, then a GW created at the first location is reinforced at the second location and the traveling GW grows according to N^2. There could, in fact, be several GWs having different "start times" or "start locations" moving up (or down) such an "array" of periodical energized GW sources.

Let us consider the IR rings (Patent Pending) in more detail. As we have calculated the IR wavelength is about 2.5×10^{-6}m. The IR waveguide has a cross-sectional area radius of λ/4 in order for it to be a monomode (lowest order mode) so that the phase doesn't change across the waveguide. Thus the cross-sectional area of each IR ring is $\pi\ (2.5\times10^{-6}\text{m}\ /4)^2 = 1.23\text{x}10^{-12}\text{m}^2$ and its diameter is 1.25×10^{-6} m. The volume of each 100-m radius toroidal ring is $2\pi\ (100)(\ 1.23\text{x}10^{-12}) = 7.7\text{x}10^{-12}\ \text{m}^3$. We divide the mass density of pentane by its molecular mass and that gives the density of jerkable masses = $6.3\times10^{28}\ \text{m}^{-3}$. Thus the number of masses in the 100-m radius circular wave guide $2N = (6.3\times10^{28})(\ 7.7\text{x}10^{-12}) = 4.85\text{x}10^{17}$ submicroscopic "particles" or potentially jerkable masses. According to Table 1 of Woods and Baker (2009) for pentane $P_i = 4.62\text{x}10^{-16}$ W. Thus the flux for all of the mass pairs in a single ring from Eq. (8) is $(1/2)(2N)(0.01146)\ P_i = 1.29\ \text{Wm}^{-2}$.

Let us next consider a more convenient laboratory arrangement for the rings. We reduce the ring radius to one meter, but set up 100 rings, concentrically (side by side in the same plane) with an average radius of the one meter. The reduced radius drops the P_i by $(100)^2$ to $4.62\text{x}10^{-20}$, but because of the 100 concentric rings the $N = 4.85\text{x}10^{17}/2$ remains the same. Thus the flux for a single "plate" of concentric rings is $1.29\text{x}10^{-4}\ \text{Wm}^{-2}$. We now stack some 10^6 of these plates on top of one another. Thus a 1.25-m high stack. In this case $n = 10^6$ and we can apply the n^2 law. Thus a HFGW total flux of $1.29\text{x}10^{8}\ \text{Wm}^{-2}$ will be generated by the stack. Of course (as R. C. Woods has pointed out, Woods and Baker (2009)) we need to be careful how much power is fed to each ring. One possible arrangement is to feed the output of one ring to the input of the next. The problem here is that the source won't have a long enough coherence length, even if the attenuation of the IR doesn't kill the power after a ring or two. To avoid this, from one source the available energizing power could be divided equally between all the rings and fed to them up the stack at the speed of light. The practical difficulties would be how to drive them all in correct phase, but it is a challenge for future research in the IR-ring approach.

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GRAVITATIONAL-WAVE AMPLITUDE

From Eq. (6A) of the Appendix of Baker, Stephenson and Li (2008) the derivation shows that the amplitude, A, of a gravitational wave is related to the flux, $F_{GW}\ \text{Wm}^{-2}$ and HFGW frequency, $\upsilon_{GW}\ \text{s}^{-1}$, by

$$A = \left(\frac{8\pi G F_{gw}}{c^3\omega^2}\right)^{\frac{1}{2}} \approx 1.28\times10^{-18}\ F_{GW}^{\frac{1}{2}}\Big/\nu_{GW} \quad . \tag{10}$$

For a 100 m radius helix the FBAR's $F_{GW} = 1.96\text{x}10^{-13}\ \text{Wm}^{-2}$ and $\upsilon_{GW} = 4.9\text{x}10^{9}\ \text{s}^{-1}$ so $A = 1.156\text{x}10^{-34}$ and through use of splitting the FBARs to increase N as discussed in Baker (2009) A can be increased by two orders of magnitude to 10^{-32}. For the 6-cm diameter helixes $A = 3\text{x}10^{-35}$ for the split FBARs. For the single IR ring $F_{GW} = 0.643\ \text{Wm}^{-2}$ and $\upsilon_{GW} = 1.2\text{x}10^{14}\ \text{s}^{-1}$ so that $A = 1.2\text{x}10^{-32}$. For the stack of rings, the amplitude of the laboratory-generated HFGWs is $A = 1.21\text{x}10^{-28}$. Thus the Li-Baker HFGW detector as configured for infrared, having a nominal sensitivity of $A = 10^{-32}$ (but possibly increased by two orders of magnitude; Li, et al., 2008 and Baker, Stevenson and Li, 2008) should be able easily to detect the HFGW generated by the stack of IR rings and also be able to detect the HFGWs generated by the single 100-m radius ring and the split FBAR-generated HFGWs on a 100-m radius helixes , but probably not on the 6-cm radius helixes. Note, however, that the fluxes and amplitudes are computed at a one-meter distance form the end of the HFGW generators and calculations in other analyses may have been much closer and give much higher values. The one-meter distance is in the far field and also compatible with the 30 cm reaction or interaction zone of the Li-baker detectors (Li et al., 2008). Also, with regard to the detectors, since the generated HFGWs are coherent and not stochastic, Eq. (3) not Eq. (4) of Stephenson (2009) applies and the sensitivity limit to HFGW detectable amplitude is greater by a factor of $\sqrt{Q} = 4.6\text{x}10^{19}$. No doubt the actual sensitivity of the Li-Baker detector can also be enhanced since the characteristics of the generated HFGW coherent signal to be detected is well known.

CONCLUSIONS

The idealized needle-shaped radiation pattern for an array of HFGW radiation elements in the far field (distances far greater than a wavelength) has been analyzed based upon the N^2 build up of large HFGW arrays including a arrays of helical form. The HFGW amplitude is reduced due to its inverse dependence on HFGW frequency and its only square root increase due to HFGW flux. The laboratory generated HFGW of a single IR-ring HFGW generator, or especially a stack of such rings, 100-m separated lines of FBARs or 100-m radius helixes of FBARs may all be detectable, but there remain problems to be solved for future research.

NOMENCLATURE

A = area of a radiation pattern cap (m^2)
A = amplitude of gravitational wave (GW) variation with time (m/m)
c = speed of light, 2.998×10^8 (ms^{-1})
f = force (N)
F = GW flux (Wm^{-2})
G = universal gravitational constant = 6.693×10^{-11} ($m^3/kg\text{-}s^2$),
n = the number of ring-plates in a stack of rings
N = number of GW radiating elements or pairs of jerking masses
P = magnitude of the power of a gravitational-radiation source (W)
r = radial distance to an object; alternately, the distance from the center of a helix to a jerking mass (m)
t = time (s)
Δ = small increment
Δf_{cf} = increment of centrifugal force change (N)
Δf_t = increment of tangential force change (N)
Δf_i = individual FBAR force change (N)
Δt = time increment (s)
λ = wavelength (m)
ν = frequency (s^{-1})
ω = angular rotational rate (rad/s)
n = number of FBAR elements in phase

Subscripts

l
EM electromagnetic
D diffraction limit
GB Gaussian beam
GW gravitational wave
i individual
t tangential
x component along the x-axis

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